Problem 1 (Syntax)
Define a formal system for the language of all binary strings that represent natural numbers divisible by 3.

Problem 2 (Semantics)
Define a formal system for the language of all binary strings of length 4. Define a semantics for this language such that the strings represent the numbers from −8 to 7. The positive numbers are those with 0 at the leftmost position. The negative numbers are those with 1 at the leftmost position. The negative numbers are represented in two’s complement notation.

The two’s complement of a binary number is defined as the value obtained by subtracting the number from a large power of two (specifically, from $2^N$ for an N-bit two’s complement).

For instance, 0100 denotes 4, 1111 denotes −1, 1000 denotes −8.

The two’s complement of the number then behaves like the negative of the original number in arithmetic (try it out with addition), and it can coexist with positive numbers in a natural way.

Problem 3
What language does the following formal system define? (The symbol $\epsilon$ is a special symbol denoting the empty string).

Problem 4
Design a formal system for the language of valid formulas in propositional logic.