1 Miniature Monopoly

Consider the miniature version of the Monopoly board from Figure ???. The rules are roughly as follows:

- Each turn, a two-sided dice is throw, and it is equally likely to come up 1 or 2.
- A token is moved anti-clockwise by the number of steps that shows up on the dice, except in the following cases:
  1. If the token lands on the square GTJ (Go to jail), it is immediately moved to the square Jail.
  2. If the token is in the square Jail and the dice shows up 1, the token is not moved, i.e., a 2 is needed to get out of jail.

Model the Monopoly board as a Markov chain to answer the following questions.

1. Give a qualitative argument to show that the token will eventually end up in Jail with probability 1.

2. In Monopoly, each time the token lands on a square, the owner of the square is paid a reward. Find the most profitable square to own and the probability of the token being in that square on the long run.

3. Assume the square marked V is Vienna. For each square, find the probability of landing in Vienna before ending up in jail.

2 Birth-Death Process

Dr. Trouble has a number of sharks living in a swimming pool. Each day, he throws a secret service agent into the pool expecting him die.

- If the agent escapes and there is at least one shark in the pool, Dr. Trouble picks a shark at random and shoots it.
- Otherwise, if the agent is killed, he adds one more shark to the pool.

Assume that each day, the agent escapes with a probability of 0.8 (For simplicity, assume that this holds even when there are no sharks in the pool). What is the average population of the sharks in the pool?
Figure 1: Monopoly board