Course: Scientific Presentation and Conduct  
Homework (till May 19)

Problem 1.

The proof of Theorem 20 on page 28 of J.L. Kelley, General Topology, 1955, is incorrect. Find the error.

Problem 2 (optional).

Assume that the following three assertions are true:

- A1. For all $x$, either $x$ is human or $x$ is a monkey, but not both.
- A2. The parent relation is well-founded; that is, there is no infinite sequence $x(0), x(1), x(2), x(3), \ldots$ such that for all $i = 0, 1, 2, 3, \ldots, x(i + 1)$ is a parent of $x(i)$.
- A3. For all $x$ and $y$, we define that $y$ is an ancestor of $x$ iff $y$ is a parent of $x$ or there exists $z$ such that $z$ is a parent of $x$ and $y$ is an ancestor of $z$.

Prove the following statement, using only proof steps from the handout ”How To Write a Proof” (lecture handout on 5 May):

G1. If there exist $x$ and $y$ such that $x$ is human and $y$ is an ancestor of $x$ and $y$ is a monkey, then there exist $x$ and $y$ such that $x$ is human and $y$ is a parent of $x$ and $y$ is a monkey.

Hint: The statement ”If there exists $z$ such that blah, then foo” is logically equivalent to the statement ”For all $z$, if blah then foo” (why?).