Homework Assignment 1 (Design techniques) 30 September, 2013

- Write the solution to each problem on a single page.
- The discussion of questions and solutions before the due date is not discouraged, but you must formulate your own solution.
- The deadline for handing in solutions is 7 October before the lecture

Problem 1. (20 = 12 + 8 points). Sort a linear array $A[1..n]$ with BUBBLESORT implemented as follows:

```plaintext
x = TRUE;
while x = TRUE do
  x = FALSE;
  for $i = 1$ to $n - 1$ do
    if $A[i] > A[i + 1]$ then
      SWAP($i, i + 1$); $x = TRUE$
    endif
  endfor
endwhile.
```

(a) Is it true that after $i$ iterations of the WHILE-LOOP the largest $i$ items are in the correct last $i$ positions of the array? Justify your answer.
(b) Is it true that after $i$ iterations of the WHILE-LOOP the smallest $i$ items are in the correct first $i$ positions of the array? Justify your answer.
(c) What do your answers to (a) and (b) imply for the running time of the algorithm?

Problem 2. (20 = 10 + 10 points). Consider distinct items $x_1, x_2, \ldots, x_n$ with positive weights $w_1, w_2, \ldots, w_n$ such that $\sum_{i=1}^{n} w_i = 1.0$. The weighted median is the item $x_k$ that satisfies

$$\sum_{x_i < x_k} w_i < 0.5 \quad \text{and} \quad \sum_{x_j > x_k} w_j \leq 0.5.$$ 

(a) Show how to compute the weighted median of $n$ items in worst-case time $O(n \log n)$ using sorting.
(b) Show how to compute the weighted median in worst-case time $O(n)$ using a linear-time median algorithm as a subroutine. Argue why your solution indeed takes $O(n)$ time.