1 Deterministic Buchi

Give an intuitive description (in plain English) of why the naive subset construction fails in determinizing the Buchi automata. Recall that subset construction is the method used for determinizing regular automata.

2 Monty Hall Problem

In the game show, Let’s make a deal, you are shown three doors. One door contains a car behind it, and the other two contain a goat. Assume for now, that you prefer the car instead of the goat. Also, assume that initially, all arrangements of the three objects are equally likely. In the first step of the game, you are asked to pick one door.

1. Monty, the host of the game knows which door contains the car. He opens a remaining door that contains a goat, and gives you the opportunity to switch your choice to the other closed door. Find the right strategy to maximize the chances of winning the car.

2. In another version, instead of Monty opening a door, a random guy from the audience with no information on the objects behind the doors runs up the stage and opens one of the remaining doors at random. You are again in the same situation as in 1, where the only open door shows a goat. Find the right strategy to maximize the chances of winning the car.

3 Nim

Consider the game of Nim played with three piles of pebbles. In each turn, Alice or Bob removes a number of stones from a single pile. The person who removes the last stone wins.

1. Draw the game graph when starting from the position with 2, 1 and 1 stones in each pile.

2. Find out which player has a winning strategy. Also, find the worst-case number of steps required for winning.

3. Find an alternate starting position with at least as many pebbles from which the other player wins.

4 Chocolates and Marzipan

You and a friend buy a bag of twelve sweets. All of them look good, except the last which contains Marzipan. And as any sane person, you do not want to eat the Marzipan. Here are the rules:

1. Your friend and you take turns grabbing some number of chocolates from the bag.

2. In each turn, either of you may take and eat 1, 2, or 3 chocolates. Anything more looks too greedy.

3. You both want to avoid eating the one that contains Marzipan.

Should you start, or do you let your friend start?