Homework 3

1. Bell numbers are the number of ways you can partition a set with \( n \) members, ignoring order. For example, a set of 3 can be partitioned in 5 ways leading to \( B_3 = 5 \).

\[
\begin{align*}
\{ \{a\} \} & \quad \{ \{b\} \} & \quad \{ \{c\} \} \\
\{ \{a\}, \{b\} \} & \quad \{ \{a\}, \{c\} \} \\
\{ \{b\}, \{a, c\} \} & \quad \{ \{c\}, \{a, b\} \} \\
\{ \{a, b, c\} \} & \quad \\
\end{align*}
\]

One way of calculating Bell numbers is to use Bell's triangle.

\[
\begin{array}{cccccc}
1 & - & - & - & - & - \\
1 & 2 & - & - & - & - \\
2 & 3 & 5 & - & - & - \\
5 & 7 & 10 & 15 & - & - \\
15 & 20 & 27 & 37 & 52 & - \\
\end{array}
\]

The above has the following Bell numbers as represented in the first column of the square matrix: \( B_0 = 1, B_1 = 1, B_2 = 2, B_3 = 5, B_4 = 15 \). Write some code that calculates the value of \( B_{21} \).

The following is an algorithm describing how to compute Bell numbers from Bell's triangle:

(a) Start with the number one and store it in the first position of the matrix.

(b) Start a new row with the rightmost (diagonal) element from the previous row as the number in the first column.

(c) For the remaining numbers in the row add the number to the left to the number above the number to the left.

(d) Repeat step (c) until there is a new row with one more element than the previous row.

(e) The numbers in the first column are the Bell numbers.