Second Homework Assignment

Write the solution to each question on a single page. The deadline for handing in solutions is Friday, October 18, 2013.

**Question 1.** (20 = 10 + 10 points). Let $X$ be a discrete set of points in $\mathbb{R}^d$, and $r > 0$. Fixing a non-negative integer, $k$, we define $R_k(X; r)$ as the set of simplices $\sigma \in 2^X$ such that

1. $\sigma \in \check{C}(X; r)$, if $p \leq k$,
2. all $(p-1)$-faces of $\sigma$ belong to $R_k(X; r)$, if $p > k$,

where we write $p$ for the dimension of $\sigma$. Note that $R_1(X; r)$ is the Vietoris-Rips complex as defined in Section 3.

(a) Is it true that $R_k(X; r) \subseteq R_\ell(X; r)$ whenever $k \geq \ell$?

(b) Is it true that $R_k(X; r) = \check{C}(X; r)$ whenever $k \geq d$?

**Question 2.** (20 = 10 + 10 points). Let $\sigma$ be an $n$-simplex.

(a) For $0 \leq k \leq n$, what is the number of $k$-faces of $\sigma$?

(b) Let $k \leq \ell$ be two non-negative integers both smaller than or equal to $n$, let $\upsilon$ be an $\ell$-face of $\sigma$, and let $\tau$ be a $k$-face of $\upsilon$. How many faces $\phi$ of $\sigma$ are faces of $\upsilon$ and have $\tau$ as a face?

**Question 3.** (20 = 10 + 10 points). Let $X$ be a finite set of points in $\mathbb{R}^d$. Recall that a minimum spanning tree of $X$ is a tree whose vertices are the points in $X$ such that the total length of the edges is a minimum.

(a) Listing the edges in a spanning tree in the order of non-decreasing length, we can define a partial order on the trees by comparing their lists lexicographically. Is it true that a minimum spanning tree is also minimal in this partial order?

(b) Let $R$ be the length of the longest edge in a minimum spanning tree of $X$. Is it true that $G(X; r)$ is connected iff $R \leq r$?