Sixth Homework Assignment

Question 1. (10 points). Let $X$ be a finite set of points in $\mathbb{R}^d$, and $r > 0$ a radius. Recall the definitions of the Čech complex and of the Vietoris-Rips complex, which we denote as $\mathcal{C}(r; X)$ and $\mathcal{R}(r; X)$. Prove that $\mathcal{R}(r; X) \subseteq \mathcal{C}(\sqrt{2}r; X)$.

Question 2. (20 = 10 + 10 points). Recall that the double-torus, $M$, is the orientable 2-manifold of genus 2. Draw $M$ in a position of your choice, and consider the height function, $f : M \to \mathbb{R}$.

(i) Draw the (extended) persistence intervals of $f$.
(ii) Draw the (extended) persistence diagram of $f$.

Question 3. (20 = 5 + 5 + 10 points). Use the (extended) persistence intervals and the persistence diagrams from Question 2 (or from the course notes if you don’t have the solution to Question 2 available).

(i) Let $r < s$ be two values. Explain how to read the rank of the image of $H_p(M_r)$ in $H_p(M_s)$.
(ii) For the same values $r < s$, explain how to read the rank of the image of $H_p(M, M_s)$ in $H_p(M, M_r)$.
(iii) Let $r$ and $s$ be any two values. Explain how to read the rank of the image of $H_p(M_r)$ in $H_p(M, M_s)$.