Homework Assignment 1 (Divide-and-conquer and prune-and-search approaches)  
7 October, 2014

- Write the solution to each problem on a single page.
- The discussion of questions and solutions before the due date is not discouraged, but you must formulate your own solution.
- The deadline for handing in solutions is 14 October before the lecture

Problem 1. (20 = 10 + 10 points). Consider distinct items $x_1, x_2, \ldots, x_n$ with positive weights $w_1, w_2, \ldots, w_n$ such that $\sum_{i=1}^{n} w_i = 1.0$. The weighted median is the item $x_k$ that satisfies

$$\sum_{x_i < x_k} w_i < 0.5 \quad \text{and} \quad \sum_{x_j > x_k} w_j \leq 0.5.$$ 

(a) Show how to compute the weighted median of $n$ items in worst-case time $O(n \log n)$ using sorting.

(b) Show how to compute the weighted median in worst-case time $O(n)$ using a linear-time median algorithm as a subroutine. Argue why your solution indeed takes $O(n)$ time.

Problem 2. (20 points). Let $X_1, \ldots, X_n$ be $\{0, 1\}$-valued random variables such that $X_i = 0$ with probability $1 - p_i$ and $X_i = 1$ with probability $p_i$. Given probabilities $p_1, \ldots, p_n$, show how to compute probabilities of events $\sum X_i = m$ for all $m \in [0, n]$, using a divide-and-conquer approach. Analyze the running time of your algorithm.

A full credit will be given for an $O(n^2)$ algorithm. (It is actually possible to achieve better complexity, but this is entirely optional).