Advanced Topics in Formal Methods
Homework 1

Due to May 13, 2015

Problem 1:

Consider the game of tic-tac-toe with player cross (X) and circle (O). Show a board configuration where it is X-player's turn to play such that the following condition holds: X-player cannot win in the next move, but there is a move of player X, such that no matter the next move of player O, in the following turn there is a move of player X to win (i.e., player X does not win immediately but can play so that after one more move player O player X can win). Show the game configuration tree, and the moves of player X to illustrate the above process.

Problem 2:

You and a friend buy a bag of twelve sweets. All of them look good, except the last which contains Marzipan. And as any sane person, you do not want to eat the Marzipan. Your friend and you take turns grabbing some number of chocolates from the bag. In each turn, either of you may take and eat 1, 2, or 3 chocolates. Anything more looks too greedy. You both want to avoid eating the one that contains Marzipan.

Recall that winning states can be computed finding the lfp of $T \cup Pre_1(X)$ from the emptyset, i.e., $X_0 = \emptyset$ and $X_{i+1} = T \cup Pre_1(X_i)$, where

$$Pre_1(X) = X \cup (\{s \in S_1 \mid E(s) \cap X \neq \emptyset\} \cup \{s \in S_2 \mid E(s) \subseteq X\}) \ .$$

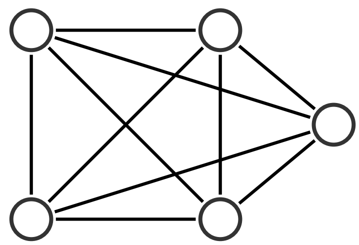
Alternatively you can compute the gfp of $T \cap Pre_2(X)$ from the whole state space, i.e., $X_0 = S$ and $X_{i+1} = T \cap Pre_2(X_i)$, where

$$Pre_2(X) = X \cap (\{s \in S_2 \mid E(s) \cap X \neq \emptyset\} \cup \{s \in S_1 \mid E(s) \subseteq X\}) \ .$$

Write the all sets X_i until you reach a fix-point, both for Pre_1 and Pre_2. Should you or should your friend start the game?

Problem 3:

Denote with K_n the graph of n nodes and all possible edges. The following is K_5.



Given K_n, you and I play a game of coloring as follows. I pick red, and you pick blue, and we take turns in coloring the edges of K_n (assume that I go first). The game ends when a triangle is formed with only one color on its edges (called a *monochrome* triangle, as opposed to a *bichrome* triangle), or no further moves are possible (i.e., all edges are colored). In the latter case the game ends in a draw, otherwise the winner is the player whose color formed the monochrome triangle.

1. Describe the game on K_6. Define the state space, identify my and your vertices in the state space, and describe the edge relation and our target sets.

2. Show that the game on K_6 cannot end in a draw. *Hint: Consider 5 edges meeting in a node of K_6. At least 3 must have the same color.* Can K_7 end in a draw?

3. **(Bonus).** A winner gets two points if he manages to form two monochrome triangles in one move. Show that on K_6, if the game has not end after 14 turns, I win two points. *Hint: Consider a complete coloring of K_6. There are $\binom{6}{3}$ triangles, and $\binom{5}{2}$ pairs of edges meeting in each node. Call such a pair bichrome, if the edges have different color. Draw a relation between the number of biochrome pairs and the number of bichrome triangles. Establish an upper bound on the number of such biochrome pairs.*