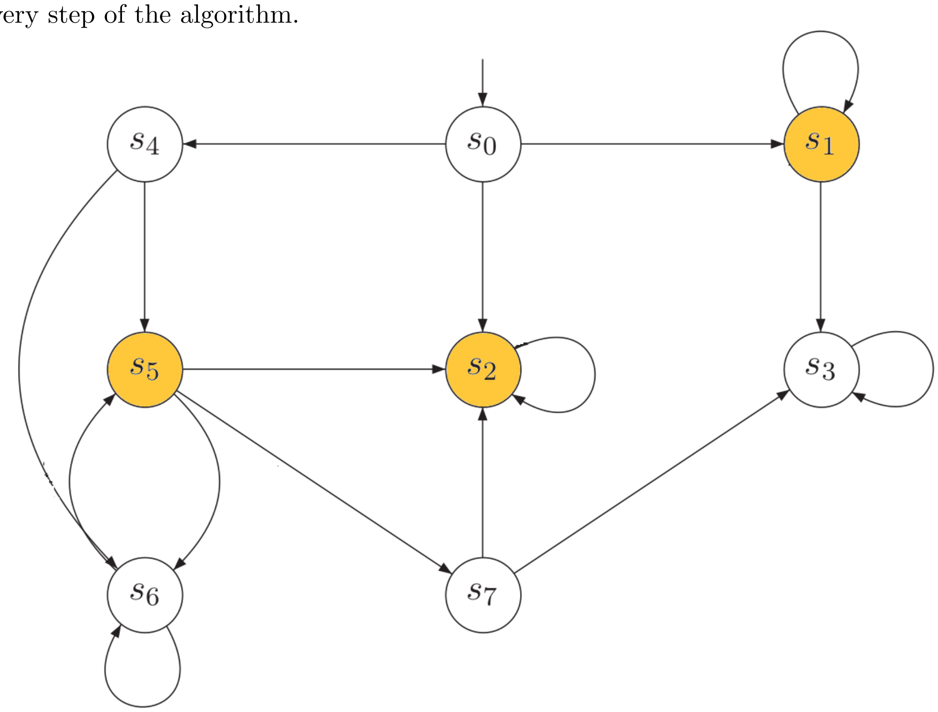
Advanced Topics in Formal Methods
Homework 4

Due on June 3, 2015

Almost-sure winning for MDPs

Consider the MDP depicted below, where orange states belong to Player 1 and the remaining states are probabilistic. The outgoing edges of probabilistic states have uniform distribution. Apply the classical algorithm to compute the set of states that almost surely can reach state s_2. Describe the states removed at every step of the algorithm.



Probability vs. non-determinism

Recall the definition of a Markov chain. Let T be a set of states. The operator $\Diamond T$ denotes all paths that reach T. The operator $\Box T$ denote all paths that stay in T. Let ϕ be one of those operators. A state s satisfies $\exists\phi$ if there exists a path rooted in s among the paths denoted by ϕ, regardless of the probabilities. A state s satisfies $\forall\phi$ if all paths rooted in s are among the paths denoted by ϕ, regardless of the probabilities. For example, the set of states that satisfy $\exists\Diamond T$ is the set of states that can reach T on the underlying graph of the Markov chain. The function $\mathbb{P}(\phi)$ denotes the joint probability of all paths rooted in s among the path in ϕ. For each of the following pairs of formulae, show or disprove whether all states satisfying the left formula on the underlying graph satisfy the right formula on the Markov chain, and whether viceversa.

(i) $\exists\Diamond T$ vs. $\mathbb{P}(\Diamond T) > 0$

(ii) $\forall\Box T$ vs. $\mathbb{P}(\Box T) = 1$

(iii) $\exists\Box T$ vs. $\mathbb{P}(\Box T) > 0$

(iv) $\forall\Diamond T$ vs. $\mathbb{P}(\Diamond T) = 1$

Recall the definition of a Markov decision process and the one a two players game. The two player game induced by the MDP is given by substituting the all probabilistic choices with non-deterministic choices of player p. On the game, a state s satisfies $\langle\langle 1\rangle\rangle(\phi)$ if player 1 has a strategy such that all paths rooted in s are in ϕ, regardless of the strategy of p, and a state s satisfies $\langle\langle p\rangle\rangle(\phi)$ if player p has a strategy such that all paths rooted in s are in ϕ, regardless of the strategy of 1. On the MDP, $\mathbb{P}^{\mathsf{max}}(\phi)$ is the maximum join probability, resp. $\mathbb{P}^{\mathsf{min}}(\phi)$ the minimum, of the paths rooted in s and belonging ϕ that can be induced by a strategy. For each of the following pairs of fomulae, show or disprove whether all states satisfying the left on the induced game satisfy the right formula on the MDP, and whether viceversa.

(i) $\langle\langle 1\rangle\rangle(\Diamond T)$ vs. $\mathbb{P}^{\mathsf{max}}(\Diamond T) = 1$

(ii) $\langle\langle 1\rangle\rangle(\Box T)$ vs. $\mathbb{P}^{\mathsf{max}}(\Box T) = 1$

(iii) $\langle\langle p\rangle\rangle(\Diamond T)$ vs. $\mathbb{P}^{\mathsf{min}}(\Diamond T) > 0$

(iv) $\langle\langle p\rangle\rangle(\Box T)$ vs. $\mathbb{P}^{\mathsf{min}}(\Box T) > 0$