# ADAPTIVE MEAN AND TREND REMOVAL OF HEART RATE VARIABILITY USING KALMAN FILTERING

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*Abstract* –**Analysis of heart rate variability requires the calculation of the mean heart rate. Adaptive methods are important for online and real-time parameter estimation. In this paper, we demonstrate the use of Kalman filtering to estimate adaptively the mean heart rate and remove the trend.** [*]
***Keywords* – Kalman filtering, heart rate variability, trend removal, adaptive estimation algorithm, non-stationarity**

## I. INTRODUCTION

Analysis of heart rate variability (HRV) requires several processing steps. After the QRS detection and the calculations of the R-to-R intervals (RRI), the mean of the RRI is removed and the spectrum is estimated. In case of an online and real-time - analysis, the estimation of the mean value must be based on past values only, since no future sample values can be used. In order to improve the computational efficiency, adaptive algorithms are appropriate for estimating the mean value of HRV.

## II. METHODOLOGY

Before Kalman filtering is introduced, we would like to review a basic adaptive algorithm. The mean can be calculated adaptively as follows:

$$\mu_k = (1\text{-}UC) \cdot \mu_k + UC \cdot y_k \tag{1}$$

y(t) is the actual sample value (e.g. the inverse R-R-Interval) at sample k; UC denotes the update coefficient and determines the degree of adaptation. In Eq. (1), only past values are used and at each time instant, the previous estimate is updated. In essence describes Eq. (1) an adaptive smoothing filter.

### *The state space model and Kalman filtering*

An alternative method to the adaptive filter is to use Kalman filtering. For this purpose, we introduce the state space model (SSM) with the system ([1] p. 102)

$$x_k = F_k \cdot x_{k-1} + w_k \qquad w_k = N(0, W_k) \tag{2a}$$

and the measurement equation

$$z_k = H_k \cdot x_k + v_k \qquad v_k = N(0,V_k) \tag{2b}$$

The properties of the state space model are determined by the matrices F and H. $u_k$ is the system input, $w_k$ and $v_k$ are uncorrelated random noise processes with covariances W and V, respectively. $x_k$ represents the (hidden) state vector, $z_k$ is the observed system output at time instant k.

Kalman filtering estimates the hidden state variable $x_k$ in a recursive way with the following equations ([1] p. 112):

State estimation extrapolation:

$$X_k(-) = F_{k-1} \cdot x_{k-1}(+) \tag{3a}$$

Error covariance extrapolation

$$P_k(-) = F_{k-1} \cdot P_{k-1}(+) \cdot F_{k-1}^T + W_{k-1} \tag{3b}$$

State estimation observational update:

$$x_k(+) = x_{k-1}(-) + K_k \cdot [z_k - H_k \cdot x_{k-1}(-)] \tag{3c}$$

Error covariance update

$$P_k(+) = [I - K_k \cdot H_k] \cdot P_{k-1}(-) \tag{3d}$$

Kalman gain

$$K_k = P_k(-) \cdot H_k^T / [H_k \cdot P_k(-) \cdot H_k^T + V_k] \tag{3e}$$

The initial conditions are determined by $x_0$ and its covariance $P_0$.

In the next step, we build a state space model for the mean heart rate. Here, we determine the best estimate of the mean heart rate. Since the unknown variable $x_k$ represents the mean estimate: $x_k = \mu_k$

There is no further information about the mean. Therefore, we assume the mean follows a random walk process. Hence, $F_k = 1$, $\mu_k = \mu_{k-1} + w_k$

The difference between the estimate $x_k$ and the actual observation $z_k$ can be assumed to be random and uncorrelated; hence $H_k = 1$.

The following update equations for the adaptive estimation of the mean can be determined as follows:

$$e_k = z_k - \mu_{k-1} \tag{4a}$$
$$\mu_k = \mu_{k-1} + K_k \cdot e_k \tag{4b}$$
$$P_{k-1} = P_{k-1} + W_k \tag{4c}$$
$$K_k = P_{k-1} / [P_{k-1} + V_k] \tag{4d}$$
$$P_k = [I - K_k] \cdot P_{k-1} \tag{4e}$$

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$V_k$ represents the variance of the prediction error and $W_k$ represent variance of the random walk. Because these parameters are usually unknown, some assumptions about their behavior can be done. As shown in [2] different assumptions can be made, including the one below (Mode a12v3):

$$V_k = (1\text{-}UC) \quad (5a)$$
$$W_k = UC^2 * I \quad (5b)$$

## III. Results

In this paper we tested the Kalman filter on both the simulated and the HRV.

### *Simulation*

Both methods, the adaptive smoothing filter and Kalman filtering, were applied to the following test series. The simulated signal has 4 epochs with 100 samples each; each epoch is a white noise process with a rms=3 and a mean value of 60, 100, 100, and 80 for the epochs 1, 2, 3 and 4, respectively. In the third epoch the mean was increased linearly from 50 to 150.

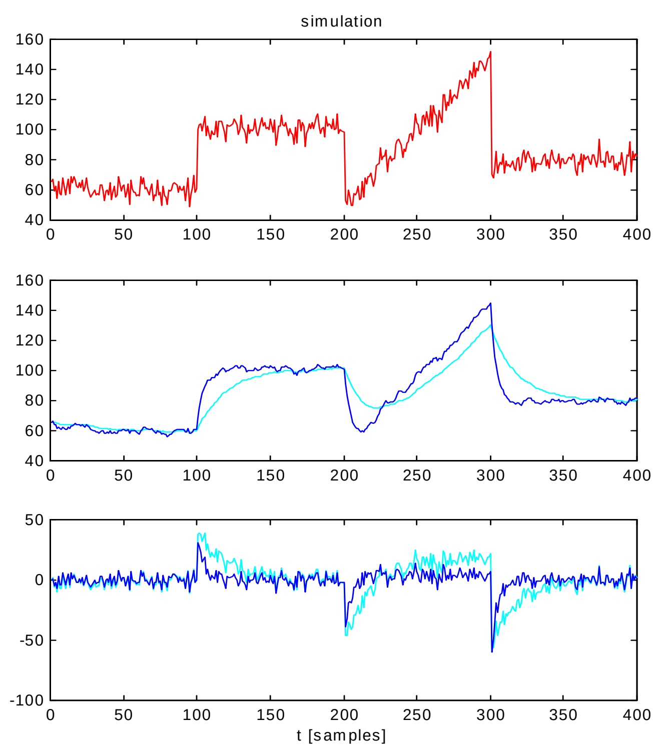


Fig 1: Simulation. The simulated data is display in the first plot. The second plots show the adaptive mean estimated with Kalman filter (dark) and exponential window (light). The third plot shows the one-step prediction error for Kalman filter (dark) and the exponential window (light). The update coefficient UC was in both cases 0.05.

The simulated signal y was analyzed using the adaptive smoothing as well as Kalman filtering. In both cases an update coefficient UC=0.05 was used. Fig 1 shows the simulated data, the estimated adaptive mean and the residual processes from both methods.

### *Heart rate variability data*

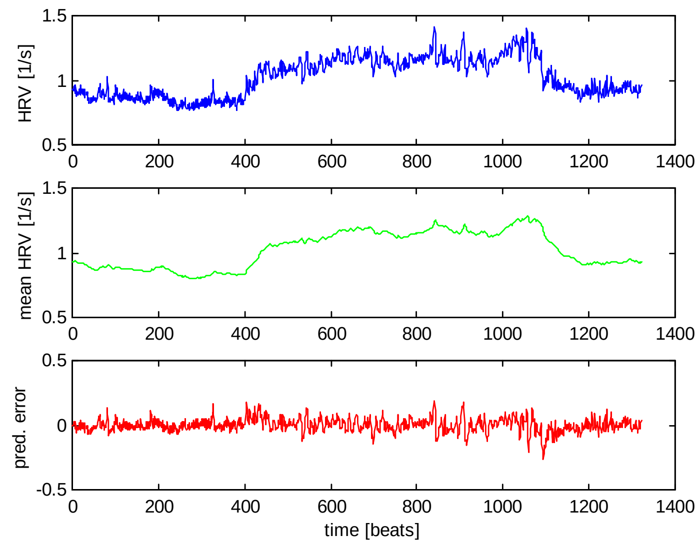


Figure 2: Heart rate variability (blue), adaptive mean (green) and the one-step prediction error (red). The prediction error is also the de-trended HRV.

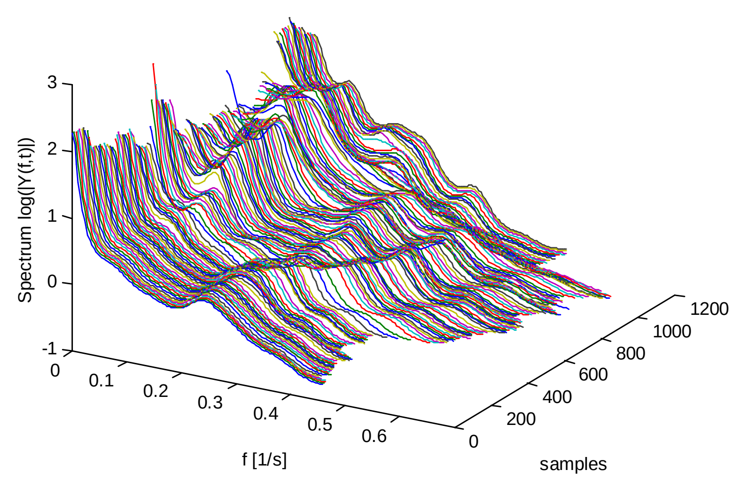


Figure 3: Time-varying spectrum based on AAR estimates of the de-trended Heart-rate variability.

In addition to the simulated data, HRV data during a tilting-table experiment was recorded (Fig. 3). Between sample 400 and sample 1100 passive head up tilt was applied to the subject. Due to passive head up tilt, the Heart Rate increased between samples 400 and 1100 from 0.9 to 1.2 Hz (54 to 72 bpm) (Fig. 2).

Our results based on 13 HRV data set showed that Kalman filtering with Mode=a12v3 and UC=0.0605 can be used to estimate the mean heart rate and the autoregressive parameters adaptively.

Fig 3 shows a time-varying spectral density function of HRV using adaptive autoregressive (AAR) parameters. Note that the Nyquist frequency changes according to the mean heart rate since the mean heart rate can be considered as the mean sampling rate of the HRV.

## IV. Discussion

In this paper we have shown how Kalman filtering can be used to estimate adaptively the time-varying mean of non-stationary time series. A comparison between Kalman filtering and exponential smoothing was done on simulated data. However, to compare both methods in an objective manner, the adaptation speed and the estimation accuracy need to be compared simultaneously. Furthermore, the optimal assumptions about the variances of the system and observation noise (Vk and Wk) must be made.

Kalman filtering was also applied to estimate the adaptive mean and the de-trended heart-rate-variability. This is important for the on-line and real-time analysis of the HRV-spectrum. The time-varying HRV-spectra (as shown in Fig. 3) can be also estimated with adaptive autoregressive methods.

Kalman filtering can be used to estimate the adaptive mean and to remove the trend of HRV data. We believe that Kalman filtering can be applied to other non-stationary biological systems, too.

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## References

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