A Hindsight on the Stream Function Solver for Liquid Simulation

Ryoichi Ando[*a], *Nils Thuerey*[b], *Chris Wojtan*[c]

[a] *National Institute of Informatics, Tokyo*
[b] *Technical University of Munich, Germany*
[c] *IST Austria, Austria*

July 27, 2020

ABSTRACT
This supplemental material describes a hindsight regarding the paper on the stream function solver for liquid simulation[1]. We clarify that our method requires future research for properly handling cases where non-negligible harmonic vector fields exist. In accordance with our hindsight, we hereby negate the algorithmic note in the original paper (Figure 1, highlighted in yellow). The detail and an example are provided below. Note that this issue is specific to our method and does not apply for a single-phased fluid[2].

> In the above exposition, we assumed that the harmonic component γ in Eq. (5) was zero, and we leave it set to zero for the simulations shown in Section 6. However, domains with non-trivial topology could be handled as described in [Elcott et al. 2007]: by first explicitly generating a harmonic vector field basis for the domain, and then projecting out and preserving the harmonic component of $\boldsymbol{u}^*$ before the stream function solve. Alternatively, we can simply treat complex boundary geometry as a dense two-way coupled rigid body, as we explain in Section 4.3.

Figure 1. Algorithmic note regarding handling cases where harmonic vector fields in the original paper, found in the last paragraph of Section 3. We illustrate an example that such a projection method does not correctly solve the motion on some cases illustrated below. Note that this problem is only specific to liquid with finite air density approaching zero (but not exactly zero). For smoke such an issue does not exist.

1 EXAMPLE

1.1 *Setup*

As a comprehensive toy example, suppose that two spherical liquid droplets are initialized both at the top and the bottom in an annular container (Figure 2). In doing so, we consider two cases: liquid droplets are large enough to block the air flow in the channel (Figure 2 left) and small enough so that the air can go though around the droplets (Figure 2 right). Note that for both cases the velocity of the droplet on the top has a positive horizontal velocity as illustrated. In this setting, a single harmonic vector field circulating in the tube is found.

1.2 *Expected motion (Ground truth)*

In case of Figure 2 left, the liquid pushes the incompressible air on the right side, so that air also pushes the liquid on the bottom, resulting in acceleration of the bottom liquid. In case of Figure 2 right, air circumvents around the top liquid, and as such the bottom liquid stays still.

1.3 *Simulated motion (Results)*

The projection method suggested in the original paper (Figure 1) can be performed either *per-component* or *globally*. Per-component indicates that we perform projection independently per topologically disconnected liquid. Global indicates that projection is performed together for all the disconnected liquids. We will review only the the latter case in this note. Although we do not examine here, the former case also produces the same result.

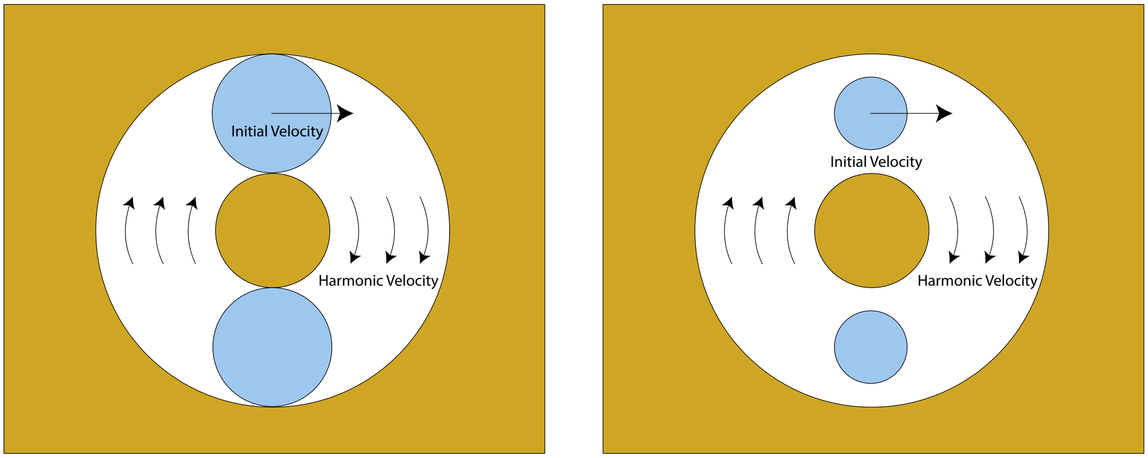

Figure 2. An example.

1.4 *Algorithm Overview*

The overview of our projection in the original paper is outlined below

1. For each harmonic vector field $\Phi_i(\boldsymbol{x})$, compute the dot-product with the velocity field

$$\alpha_i = \int \Phi_i(\boldsymbol{x}) \cdot u(\boldsymbol{x}) dV. \quad \textbf{Equation 1.}$$

2. Subtract it from the velocity field $u^\star(\boldsymbol{x}) = u(\boldsymbol{x}) - \sum_i \alpha_i \Phi_i(\boldsymbol{x})$.
3. Let $P(\cdot)$ be a projection operator using a stream function solve. Project $u^\star(\boldsymbol{x})$ and recover the harmonic components, that is

$$u^{\text{new}}(\boldsymbol{x}) = P\left(u^\star(\boldsymbol{x})\right) + \sum_i \alpha_i \Phi_i(\boldsymbol{x}). \quad \textbf{Equation 2.}$$

If we apply our projection method, we obtain a single non-zero alpha coefficient. Hence, the $u^\star$ on the bottom will have a non-zero velocity. Since the harmonic vector fields are divergence-free by construction, our projection operator has no effect on the bottom liquid. Consequently, the velocity on the bottom u^{new} will be recovered to zero. Therefore, our projection method does not correctly handle the case of Figure 2 left.

1.5 *Discussion*

We have not yet found an algorithmically consistent solution to correctly handle arbitrary cases. At the moment, our method[1] cannot guarantee physically accurate animations in the presence of non-trivial harmonic vector fields. We believe the problem boils down to an unclear definition of the projection operation at the liquid-gas interface. As the projection operation is well-defined for incompressible fluid with variable density, we hope that a more careful projection operation that takes into account both the gas and liquid phases will be able to resolve the issue with general multiphase fluids in the future.

REFERENCES

1. Ryoichi Ando, Nils Thuerey, and Chris Wojtan. A stream function solver for liquid simulations. *ACM Trans. Graph.*, 34(4), July 2015.
2. Sharif Elcott, Yiying Tong, Eva Kanso, Peter Schröder, and Mathieu Desbrun. Stable, circulation-preserving, simplicial fluids. *ACM Trans. Graph.*, 26(1), January 2007.