An Algorithmic Approach to Global Asymptotic Stability Verification of Hybrid Systems

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### Stability
- Stability is a fundamental property in control system design
- It captures the property that small perturbations to the input of a system result in only small deviations in the behaviour of the system

#### Classical notions of stability
- Lyapunov Stability (LS)
- Asymptotic Stability (AS)
- Global Asymptotic Stability (GAS)
- Region Stability (RS)

Global asymptotic stability ensures that the system converges to the equilibrium point starting from any state of the system.

### Asymptotic stability verification

**Lyapunov stable** • A system is LS with respect to 0 if for every ε > 0 there exists δ > 0 such that for every execution σ starting from B(δ), σ ∈ B(ε).

**Asymptotically stable** • A system is AS with respect to 0 if it is Lyapunov stable and there exists a value ζ > 0 such that every execution σ starting from B(ζ) converges to 0.

#### Algorithmic approach

Reduce the problem to graph analysis problem:

- Quantitative predicate abstraction of a hybrid system \( H \) into a abstract weighted graph \( A(N, F) \)
- Analyse \( A(N, F) \) for absence of cycles with weight greater than 1.

#### Stability zone computation

\( Z \subset \mathbb{R} \) is a stability zone with respect to \( \mathbb{R} \) if every execution starting in \( Z \) will remain in \( \mathbb{R} \).

**Construction steps**
- Extract the central region from the hybrid system \( H \)
- Compute the maximum scaling \( M \) associated with the paths in the abstract weighted graph
- Shrink the central region by a factor of \( M \) to obtain the stability zone

### Current Research

- Extension of the algorithmic stability verification to non-linear systems.
- Compositional analysis for input-output stability verification.
- Synthesis of state based switching control for a family of dynamical systems.

### Publications

- Countersample Guided Abstraction Refinement for Stability Analysis, Pavithra Prabhakar and Miriam García Soto, CAV 2016
- An algorithmic approach to stability verification of polyhedral switched systems, Pavithra Prabhakar and Miriam García Soto, ACC 2014
- Abstraction Based Model-Checking of Stability of Hybrid Systems, Pavithra Prabhakar and Miriam García Soto, CAV 2013

**Link:** [http://software.imdea.org/projects/averist/](http://software.imdea.org/projects/averist/)