SYNERGIES AND VISTAS IN ANALYTIC NUMBER THEORY (3–7 SEPTEMBER, 2012)

Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
0900–0930	*Registration*				
0930–1030	A. Gorodnik	R. de la Bretèche	B. Green	D. Masser	K. Soundararajan
1030–1100	*Coffee*	*Coffee*	*Coffee*	*Coffee*	*Coffee*
1100-1200	B. Conrey	H. Iwaniec	J. Brüdern	A. Kontorovich	G. Harcos
1200–1400	*Lunch*	*Lunch*	*Lunch*	*Lunch*	*Lunch*
1400-1500	A. Granville	P. Michel	A. Cojocaru	C. David	
1500–1530	*Coffee*	*Coffee*	*Coffee*	*Coffee*	
1530–1630	P. Sarnak	E. Kowalski	M. Young	T. Wooley	
1700–1800	*Contributed talks* 1700–1730: F. Thorne 1730–1800: P. Vishe	*Contributed talks* 1700–1730: J. Van Order 1730–1800: A. Södergren		*Contributed talks* 1700–1730: C. Elsholtz 1730–1800: J. Stopple	
Evening	1830 : *Apéro* St. John's College		1900: *Conference drinks* 2000: *Conference dinner* New College		

Invited talks

Régis de la Bretèche, *Density of Châtelet surfaces failing the Hasse principle*

Abstract: Châtelet surfaces provide a rich source of geometrically rational surfaces which do not always satisfy the Hasse principle. We investigate the frequency that such counter-examples arise over the rationals. This is joint work with T. Browning.

Jörg Brüdern, *On the Möbius function*

Alina Cojocaru, *Frobenius fields for elliptic curves*

Abstract: Let E be an elliptic curve defined over $\mathbb{Q}$. For a prime p of good reduction for E, let π_p be the p-Weil root of E and $\mathbb{Q}(\pi_p)$ the associated imaginary quadratic field generated by π_p. In 1976, Serge Lang and Hale Trotter formulated a conjectural asymptotic formula for the number of primes $p < x$ for which $\mathbb{Q}(\pi_p)$ is isomorphic to a fixed imaginary quadratic field. I will discuss progress on this conjecture, in particular an average result confirming the predicted asymptotic formula. The latter is joint work with H. Iwaniec and N. Jones.

Brian Conrey, *A q-analogue of the Balasubramanian, Conrey, Heath-Brown conjecture*

Abstract: In joint work with Iwaniec and Soundararajan we use the asymptotic large sieve to obtain an asymptotic estimate for the average over q and over primitive characters modulo q of the mean square of the the central value of the Dirichlet L-function multiplied by a Dirichlet polynomial of length nearly q. As an application we can prove that in a certain average sense at least $3/5$ of the zeros of Dirichlet L-functions are on the critical line.

Chantal David, *Elliptic curves with prescribed groups over finite fields and Cohen–Lenstra Heuristics*

Abstract: Let $G_{m,k} := \mathbb{Z}/m\mathbb{Z} \times \mathbb{Z}/mk\mathbb{Z}$ be an abelian group of rank 2 and order $N = mk^2$. When does there exist a finite field $\mathbb{F}_p$ and an elliptic curve $E/\mathbb{F}_p$ such that $E(\mathbb{F}_p) \simeq G_{m,k}$? More precisely, let

$$S(M,K) = \{m \leqslant M, k \leqslant K \,:\, \exists p,\, E/\mathbb{F}_p \text{ with } E(\mathbb{F}_p) \simeq G_{m,k}\}\,.$$

It was conjectured by Banks, Pappalardi and Shparlinski that

$$\#S(M,K) = \begin{cases} \underline{o}(MK) & \text{if } K \ll (\log M)^{2-\varepsilon} \\ MK(1+\underline{o}(1)) & \text{if } K \gg (\log M)^{2+\varepsilon}. \end{cases}$$

We prove in this talk that the first part of their conjecture holds for the whole range $K \ll (\log M)^{2-\varepsilon}$, and that the second part holds for the limited range $K \geqslant M^{1/4-\epsilon}$. We also show that $\#S(M,K) \gg MK$ holds for a larger range. The fact that the groups $G_{m,k}$ are more likely to occur when m is small is reminiscent of the Cohen–Lenstra heuristics which predict that a random abelian group G occurs with probability weighted by $\#G/\#\mathrm{Aut}(G)$. In order to see that the probability of occurrence of the groups $G_{m,k}$ is really weighted by the factors $\#G_{m,k}/\#\mathrm{Aut}(G_{m,k})$, one should count the number of times that a given group $G_{m,k}$ occurs over the finite fields $\mathbb{F}_p$, and not only when a given group occurs. Let G be a group as above of order N, and let

$$M_p(G) = \#\left\{E/\mathbb{F}_p \;:\; E(\mathbb{F}_p) \simeq G\right\}.$$

We show that, under a suitable hypothesis for the number of primes in short intervals, we have for $\#G/(\log(\#G))^A \ll \exp(G)$

$$(\star) \qquad \frac{\log N}{4\sqrt{N}} \sum_{N+1-2\sqrt{N}<p<N+1+2\sqrt{N}} M_p(G) \sim \frac{K(G)\#G}{\#\mathrm{Aut}(G)} N^{3/2},$$

where $K(G)$ is non-zero and uniformly bounded. Finally, we discuss how to obtain some unconditional upper bounds, and unconditional lower bounds for most of the groups G, for the average of $(\star)$ which exhibit the Cohen–Lenstra weights. This is joint work with V. Chandee, D. Koukoulopoulos and E. Smith.

Alex Gorodnik, *Diophantine approximation on homogeneous varieties*

Abstract: We discuss quantitative density of rational points lying on algebraic varieties. In many cases the set of rational points can be parameterised using actions of algebraic groups. Then this problem can be explored using dynamical systems and harmonic analysis techniques. We outline an approach to Diophantine approximation on homogeneous varieties based on ergodic theorems with explicit rates determined by the spectral gap property of automorphic representations. This is joint work with A. Ghosh and A. Nevo.

Andrew Granville, *Beyond heuristics: finding out when the sieve works using additive combinatorics*

Abstract: In joint work with Koukoulopoulos and Matomaki, we explore what happens when we sieve an interval of length x with many primes $> x^{1/2}$.

Ben Green, *The inverse large sieve problem*

Abstract: Start with $\{1, \ldots, N\}$ and throw away $(p-1)/2$ of the residue classes modulo p for every prime $p > 2$. One can be left with a moderately large set: if one dispenses with the quadratic non-residues then the set of squares, which has cardinality about $N^{1/2}$, remains. This is essentially sharp by the large sieve. Helfgott and Venkatesh asked whether any situation in which equality nearly occurs comes from a quadratic example like this. We cannot get even vaguely close to their conjecture, so we instead explore some consequences of it and prove that one can beat the $N^{1/2}$ bound in certain specific cases when the set of residues has combinatorial structure. This is joint work with A. Harper.

Gergely Harcos, *On the sup-norm of Maass cusp forms of large level*

Abstract: Let f be a Hecke–Maass cuspidal newform of square-free level N and Laplacian eigenvalue λ. I will discuss the recent joint result with Nicolas Templier that $\|f\|_\infty \ll_{\lambda,\epsilon} N^{-1/6+\epsilon}\|f\|_2$ for any $\epsilon > 0$, with an implied constant depending continuously on λ.

Henryk Iwaniec, *Exceptional discriminants*

Abstract: This will be about things which entertain or bother us regardless whether they do exist or not.

Alex Kontorovich, *Integral Apollonian gaskets*

Abstract: We will present recent work with Jean Bourgain on the local–global problem for integral Apollonian gaskets.

Emmanuel Kowalski, *Algebraic twists of modular forms II*

Abstract: This is the second part of a two-part talk shared with Philippe Michel.

David Masser, *Torsion points on group surface schemes and applications*

Abstract: Recently with Umberto Zannier as well as Daniel Bertrand, Pietro Corvaja and Anand Pillay we have practically completed a programme about torsion points on commutative group surface schemes in the context of unlikely intersections in zero characteristic. We describe the main results together with applications to Pell's Equation over polynomial rings and the integration of algebraic functions in elementary terms.

Philippe Michel, *Algebraic twists of modular forms I*

Abstract: We consider estimates for sums of Fourier coefficients of modular forms twisted by functions of "algebraic origin". Using the amplification method and the Riemann Hypothesis over finite fields, in particular the Deligne–Laumon theory of the Fourier transform, we obtain very general estimates for such sums. This also has applications to the equidistribution of similarly twisted Hecke orbits. This is joint work with É. Fouvry and E. Kowalski.

Peter Sarnak, *Thin matrix groups and diophantine analysis*

Kannan Soundararajan, *Moments of L-functions*

Trevor Wooley, *Weyl sums and efficient congruencing*

Abstract: We report on the latest developments in the efficient congruencing method. Thus far, the method has delivered estimates for Vinogradov's mean value theorem within a factor 2 of the best possible in two directions, superseding previous work by a logarithmic factor. Further improvements are forthcoming, and also new applications of the method.

Matt Young, *L^4-norm of cusp forms of large weight.*

Abstract: I will discuss recent work with V. Blomer and R. Khan on the concentration of mass of holomorphic Hecke cusp forms of large weight, as measured in different ways.

Contributed talks

Christian Elsholtz, *Notes on the divisor function and some of its applications*

Abstract: In joint work, we recently studied the maximum order of the iterated divisor function and proved an asymptotic for $\log d(d(n))$. This problem goes back to Ramanujan (1915). Also, the number of solutions of m/n as a sum of unit fractions is largely determined by certain divisor functions. We report on some of these results.

Anders Södergren, *On Epstein's zeta function and related results in the geometry of numbers*

Abstract: In this talk I will present results on the value distribution and zeros of the Epstein zeta function $E_n(L, s)$ for a random lattice L of large dimension n. Some of the key ingredients in our discussion are of independent interest in the geometry of numbers; in particular I will describe a new bound on the remainder term in the generalized circle problem.

Jeffrey Stopple, *Repulsive behaviour in an exceptional family*

Abstract: The existence of a Landau–Siegel zero leads to the Deuring–Heilbronn phenomenon, here appearing in the 1-level density in a family of quadratic twists of a fixed genus character L-function. We obtain explicit lower order terms describing the vertical distribution of the zeros, and realise the influence of the Landau–Siegel zero as a resonance phenomenon.

Frank Thorne, *Counting cubic fields using analytic number theory*

Abstract: For any $X \geqslant 1$, let $N_3(X)$ be the number of cubic fields K such that $|\mathrm{Disc}(K)| < X$. The Davenport–Heilbronn theorem states that $N_3(X)$ is asymptotic to $\frac{1}{3\zeta(3)}X$, and I will discuss my ongoing work with Takashi Taniguchi on refining this estimate. I will first recall our proof of the existence of a secondary term of order $X^{5/6}$. I will then discuss some more recent related work. I will describe how our methods allow us to count Galois sextic fields with Galois group S_3, but I will exhibit some numerical data that shows that we haven't managed to tell the whole story (even with a secondary term). I will also discuss how our methods can be combined with those of Bhargava, Shankar, and Tsimerman, yielding a simplified proof with better error terms.

Jeanine Van Order, *Some generalizations of Mazur's conjecture*

Abstract: Iwasawa main conjectures, at least in the context of the conjecture of Birch and Swinnerton-Dyer, often suggest the generic nonvanishing of large families of special values of L-functions. I will describe in this talk the setting suggested by the so-called two variable main conjecture for non-CM elliptic curves, which applies to Rankin-Selberg L-functions in the general, non self-dual setting.

Pankaj Vishe, *Cubic hypersurfaces and a version of the circle method over number fields*

Abstract: A version of the Hardy-Littlewood circle method is developed for number fields K and is used to show that any non-singular projective cubic hypersurface over K, with dimension at least 8, always has a K-rational point. This is joint work with T. Browning.